TECHNOLOGICAL DEVELOPMENT IN GOTHIC VAULTS DESIGN

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1.- INTRODUCTION

1.1. - Evolution of Gothic architecture

The Gothic was an architectural style which achieved to generate an aesthetic code perfectly articulated. However, beyond its stylistic singularities, it is interesting to note how the composition rules that made up this style are explained by the presence of a vector which, relentlessly, impelled the Gothic to reach higher and higher complexity levels. With regards to the construction of vaults, it is a well-known fact that the Gothic style represents the success of the ribbed vault which is, undoubtedly, one of the most revolutionary inventions in the history of construction. The history of the Gothic vault is a long one which starts with the simple cross ribbed vaults of the beginning of the 13th century to the complex and spectacular late Gothic vaults of the 16th century.

The principle of complexity as the evolution vector of the Gothic style was an idea largely developed in the middle of the last century by Paul Frankl. This Communication tries to complete this idea by adding that the formidable complexity reached by the European Gothic could be attained thanks to the tools generated in the workshops of the medieval stonemasons masters: geometry and standardization. In short, it is about reaching as much complexity as possible, but with the higher standardization, so that the most complicated ribbed vault can be finally built with just one arch. The advantages of this technique are obvious: all the voussoirs with which the arch ribs of the vault are built are equal and the wood centrings which will help its construction are identical.

1.2. - From the semi-circular arch to the ogival arch

One of the distinctive traits of Gothic architecture from its start was giving up of the semi-circular arch and its replacement by the pointed arch. The widespread use of the pointed arch, beyond constituting an essential stylistic feature, means a noticeable technological evolution.

Let's see some of the advantages of building with pointed arches. If the aim is to cover a certain span with a semi-circular arch, the solution is unique: the voussoirs with which the arch have to be built will be determined by the radius of curvature of the circumference which the arch describes. If, in the same piece of work, another arch with a different span is required, the voussoirs needed for this arch would have a different radius of curvature and would, therefore, be different from those of the previous arch and, likewise, that would, therefore, be the case with all the arches of different spans. Consequently, when building with semi-circular arches, the organization, storage and assembling of the voussoirs is extraordinarily complicated, since the pieces are not interchangeable; each one requires its appropriate position. Nevertheless, in order to cover the same span, instead of a semi-circular arch, the choice was to use a pointed arch, the solution is indeterminate, that is, there are infinite
pointed arches that could cover a certain span. That is why it is necessary to define another parameter which determines the solution to use. The most usual parameter used in medieval times was the height of its keystone; this piece of information determined the radius and the centre of curvature, assuming that the arch starts with vertical tangency. This degree of indetermination inherent to geometry of the pointed arch is, at the same time, one of its most powerful advantages. If the plan is to build another arch to cover a different span, it is possible to draw another figure with the same radius of curvature, and, in this case, the variation occurs in the height of the keystone. It is, therefore, possible to draw several arches to cover different spans with the same radius of curvature or, in other words, the same voussoirs can be used for different arches. Figure 1 shows a drawing by Viollet-le-Duc in which this principle is summarized: with the same curvature of a semi-circular arch several pointed arches can be drawn. Applying this principle to a star-shape ribbed vault in which the diagonal arch is a semi-circular arch, the other arches, the tierceron and the formeret could be built with identical curvatures.

Therefore, the use of the pointed arch entails a remarkable advantage for the organization of the works and the transmission of orders among the different members of the workshop constructing the building.

2. - THE STANDARDIZATION OF THE VAULT

The evolution of the Spanish Gothic architecture in its latest period, in the middle of the transition between the Late Middle Ages and the modern era, is marked by the development of ribbed vaults more and more complex in which the ribs and the boss which form them keep multiplying incessantly. The most usual model is the stellar groin vault, formed by five bosses (a central boss and four secondary). This composition works as the base of a great number of a lot more complex compositions in which both the keystones and the tiercerons increase in number to attain compositions more and more spectacular. As a general rule, all these vaults use compass drawn arches, that is, with a unique centre located in the impost line of the vault, but the vaults drawn with arches having three centres, i.e. ovals, are also very frequent.

The strategies to make all the arches with which the vault will be built be equal are summarized up in the four sketches shown in figure 2, that could be described as follows:

A. - All the arches are equal because they coincide with the curvature of the diagonal.
B. - The arches are equal because the diagonal arch (semi-circular) bends forward until it reaches the height of the secondary keystones previously determined.
C. - When using oval arches, the upper portion of the oval can slide down the lower part. The point of tangency between the two curvatures of the oval moves, but all the arches can be built with the same curvatures.
D. - The previous drawing can be simplified even more. The lower part of the oval arch does not really exist, it is the solid springing of the vault. The upper part of the oval starts always at the same height, where the springing finishes. From this point, as in case B, the arch formed by the upper part of the diagonal oval tilts to reach the heights of the boss.

Let’s see now with more detail the application of each one of these cases.
2.1. - Vaults in which all the arches have the same curvature

When the vault’s ground plan is square, if we draw the diagonal arches semi-circular and, then, with the same radius of curvature we design the four perimeter arches, the boss stones of these arches reach a height slightly lower than the central keystone. It is, therefore, a vault in which the ridge line is practically horizontal and in which the ogival and formeret arches have the same curvature. Besides, if we draw the tiercerons with the same radius, we manage to build a vault in which all the arches have the same curvature (figure 1). This special feature, which could simplify so much the cutting of voussoirs and the making of centrings, made very common this type of vaults.

Let’s see the application of this principle in the vault that covers the arms of the transept in Astorga cathedral (figure 3). This vault, attributed to Rodrigo Gil, must have been built between 1550 and 1570. Its plan is square with a side 7,58 m long and its central keystone is 5,14 m high over the impost plan. It has a couple of tiercerons in each direction and the subsidiary ribs describe two figures tangent to each other: four lobes formed by segments of circumferences in the centre and a convex quadrilobe, that is, with its curvature to the outside, ending in ogees. This design could be considered typical of the Hontañón family and, with more or less variations, duplicated in many Castilian churches.

As shown in figure 4A, the tiercerons are located in the bisector of the angle formed by the diagonal and the formeret, the rest of the bosses come from the superposition of a 4x4 net over the plan; this grid fixes point 1, with which the central lobes are set, as well as the tangency point 4. The circumference with its centre in 5 is designed with the same circumference with which the central lobe is drawn.

When carrying out the geometric reconstruction of arches in figure 4B, it can be observed that the diagonal arch is a perfect circumference surmounted 15 cm at the springing level. With the real measures of the vault, its elevation has been reconstructed and one notices that the height of the boss of the tiercerons 2 and the formerets 3 coincides with the path of the diagonal arch (points 2 and 3). The same happens for the secondary bosses A, B and C. The result is a vault in which its ridges, the rampantes, are nearly horizontal, barely a slight inclination from the central keystone 1 to the keystones of formerets 3. Hence, the formerets and tiercerons are pointed arches formed with segments of the semi circumference of the diagonal arch. Figure 5 shows a three-dimensional reconstruction of this vault in which the shape obtained as a result of designing the vault according to these principles can be noted and its ridge lines are nearly horizontal.

2.2. - Vaults with tilted arches

At the northwest angle of the cloister of Segovia cathedral is located a vault built by the architect Juan Guas, which stands outs for the originality and singularity of its composition (figure 6). At first glance, the lack of symmetry in all its plans is remarkable; the symmetry only exists in the vertical plans containing the diagonal arches. In addition, there is no central keystone, since the ogival arches compose a central rhombus, in which have been carved as folds of the filling shell the geometrical lines where the ribs should continue. This singular design can be observed in some medieval German sources, such as the collection of drawings “Wiener Sammlungen” or in the “Frankfurt Notebook”(BÖKER, 2005. BUCHER, 1979).
However, once again, the versatility of the system enables to easily obtain the layout of this vault (figure 7). Although physically they are not complete, the diagonal arches form a semicircle which determines the height of the central point of the vault. Once drawn the arches, the architect defines the form of the vault, which is determined by its two cross sections. Both lines, the longitudinal and the transverse ridge lines define the “fall” of the vaulted surface toward the formerets and transverse arches and, also, determine the height of the boss located above them.

With these premises, the diagonal arches, tiercerons and formerets should be different, each one with its particular curvature. However, a more detailed examination reveals singular resources so that all the ribs can be drawn with only one curvature. Let’s see next how this objective is achieved (figure 7). First of all, the radius of the diagonal arch is determined by the semi-diagonal of the square (since this arch is a semi circumference), and, likewise, the height of the central keystone. It’s now time to draw the tierceron, point 3, located at the height fixed by the ridge line; the same diagonal arch is used to design this arch, just by tilting it over until it reaches height 3. Obviously this arch will have its centre below the imposts line and, in its origin, does not start tangent to the vertical line to the impost level; it forms a small dodge at the start which is almost not noticeable. The procedure for the formeret is the same; again, from its springing we pivot the diagonal arch until it reaches height 4; the arch, in the impost level, starts again inclined with regard to the vertical and its centre is located below the imposts line.

Therefore, in short, we could say that this method to standardize curvatures proves to be opposite to the one described previously. In the first case, the shape of the vault was a consequence of using the same diagonal arch for the rest of the arches; the result was invariably a flat ridge line vault. With the method just described, the form of the vault is determined first; that means that the vault will adopt any form deemed appropriate; subsequently, the heights of its main bosses will be obtained when drawing its ridge lines; one has just to tilt the diagonal arch to reach the height of all the bosses. In the Departamento de Construcción of the Escuela de Arquitectura de Madrid, we had the opportunity to verify the hypotheses aforementioned, through the construction of a model scale 1:3 of Juan Guas’ vault (figure 8). This experience is a firm confirmation of this construction method.

The results of these skilful resources are obvious: all the arches have the same curvature, or, in other words, the whole vault is built with only one arch, and, therefore, all the voussoirs are equal. There is no need to insist on the advantages this implies for the organization and carrying out of the works. The search for the standardization and systematization is clearly proved in this case, even though the geometric design is really sophisticated and singular.

2.3. - Vaults with oval arches

Among the new vaults developed during the late Spanish Gothic style are the oval star-shape ribbed vaults. These are vaults in which the diagonal arches, instead of semi circumferences, are arches with three centres: ovals. In principle, they could be described as depressed vaults, although sometimes, at their springing level, they can be surmounted and reach a similar height in the central keystone to that of a regular vault. In this type of vault only the diagonal and the transverse arches are complete ovals, the curvatures of the rest of the arches, tiercerons and formerets, are usually a portion of the half part of the oval arch that described by the diagonal. Therefore, by symmetry, these arches become pointed arches with four centres: Tudor arches.
They have certain unquestionable advantages; first of all, as we have just mentioned, they are depressed vaults, which means that, by reaching a lower height than a traditional vault, the general height of the building is limited and so is its cost; secondly, they are vaults with a surface relatively flat at their culmination, which makes possible certain more complex designs, such as concentrical circles difficult to carry out in a traditional vault. In addition, the use of oval arches allows the standardization to a great extent of arches since, as figure 9 shows, once the oval of the diagonal with its centres C1 and C1' is drawn, the tierceron 2 can be drawn also with the same curvature: the lower part of the oval, with its centre in 1, and the upper one, sliding the upper curvature of the oval from the tangency point t to t'. This clever use of the two oval fragments enables the rest of the ribs that form the vault to be solved with the two curves with which the diagonal oval is composed.

Let’s analyse a case of this type of vaults carried out by one of the most creative and surprising architects of Spanish Gothic: the vault the architect Juan de Álava built in the cloister of San Esteban monastery, in Salamanca, around 1533. It is a vault apparently simple but whose design reveals unsuspected surprises (figure 10).

It’s in fact a square vault with five keystones. The tiercerons (2), as shown in figure 11A, are placed in the diagonal’s position, in the middle of the angle between the diagonal arch and the transverse arch, or between the diagonal and the formeret. The form in which the position of this bosses was fixed is well known, once designed the circumscribed circumference and the straight line 1-0 which, when crossing with the axis, determines the length 2-3, that is, the distance separating the tierceron’s bosses from the formeret’s one. Lastly, it has four subsidiary arches in circumference quarters whose measurement seems to be determined by the alignment A-B-C; this is an open subsidiary rib, that is, destined to connect with that of the adjoining vault and to cause a linking of ribs.

When designing the arches (figure 11B), we immediately realize that the diagonal arch’s keystone is below the theoretical semi-circular arch; this arch is, therefore, a depressed arch which has to be traced with three centres. The oval 1, with centres C1 and C1’, which seems to adjust accurately to the real arch, was designed with the measurements that could be taken in situ.

Next, we go on to draw the tiercerons, taking into account that, imperatively, they must reach the height determined by the horizontal ridge line of this vault. It’s obvious that the tierceron and the formeret could be pointed arches different from each other, but it’s a lot more advantageous to build them with the same diagonal arch, and so make the three arches forming this vault have equal curvatures. To carry out this operation, one has just to make the upper part of arch 1 roll over its lower part 15º; this operation makes the keystone 1 move to position 2, and the point of tangency of the oval 1 move to 2. The keystone of formeret 3 is done in the same manner, now making the upper part of oval 4 roll over the lower part. Three arches are obtained this way, different but nonetheless with the lower and upper part in common. This method simplifies in a large way the cut of voussoirs.

The vault, because it’s depressed, has a flattened ridge profile which allows to place the circular leaf in its top managing that this subsidiary rib describes its circumference without many fragmentations. The same happens with the corner’s subsidiary rib which, thanks to the fact that the three arches of the vault are the same oval, can describe a continuous and horizontal circumference. Finally, let’s point out that the vault covers its filling cells in a French way, that is, as a groined vault. The three-dimensional image of a portion (Fig. 12) allows a clearer display of the resulting volume, and we can see its ridges strictly horizontal and the oval diagonal arch.
2.4. - Vaults with inclined oval arches

To illustrate the geometric principles of this new way to project a vault, we will consider the vault of the elevated choir of San Esteban convent in Salamanca (figure 13), also built by architect Juan de Álava around 1520. As shown in figure 14, it is a vault strongly depressed, a flat vault achieved thanks to the use of oval arches of little elevation. It is a rectangular plan, whose approximate dimensions are 15x7.5 meters, as shown in figure 15, in which we also discover its complex plan and the ridge of its two sections: the transverse, bent towards the formeret, and the longitudinal, straight and horizontal.

Next, Juan de Álava defines the ridge lines of the vault, that is the longitudinal and transverse curvatures. For the longitudinal ridge (in the direction of the nave’s axis) he determines a straight and horizontal line, that means that the height of the central keystone must be the same as that of the transverse arch; on the contrary, in the transverse direction, he fixes a slightly curved ridge line with a slope towards the formeret’s keystone. These two lines are essential since they define with precision the shape of the vault and, in addition, they fix the height of the secondary bosses.

To determine the curvature of the tiercerons, the architect seems to have used the strategy summarized in the sketch in figure 16. First of all, the lower part of the oval, which is not strictly an arch, coincides exactly with the springing of the vault and its construction is done in horizontal layers. These horizontal courses reach the height of the point of tangency between the upper and lower curvatures of the diagonal oval. This oval had been traced with the height of the central keystone (point 1) that the architect deemed appropriate. Secondly, the architect determines that all the tiercerons have to be oval arches with identical curvatures to that of the diagonal arch. Next, two conditions that will simplify extraordinarily the construction of the vault are set. First, the lower part of the tiercerons’ oval is made to coincide with that of the diagonal arch, and then the height of the springing which fixed the diagonal arch, H, is determined so as to be identical for all the arches (see again figure 16). With these conditions, the different tiercerons are traced easily, making the upper part of the diagonal arch rotate from the point H until it coincides, on the other end, with the height of each one of the bosses. In short, the vault has been built with only one arch: the upper part of the oval of the diagonal arch.

In Madrid’s Escuela de Arquitectura, there was a chance to test the hypotheses exposed in the previous paragraphs by means of the construction of a model of this vault; the experiment allowed to verify the certainty of the previous considerations. To carry out the vault, in the first place, its layouts were drawn, that is its real size design, in plan and elevation. Following the patterns we know from the medieval stonemasonry workshops, the information which allowed to undertake the cutting of each one of the pieces that form the vault (figure 17) was extracted from these drawings (figure 17). Once the cutting process was finished, the setting up of the vault started by placing the pieces previously cut over wooden centrings (figure 18). Obviously, the centrings, all of them with only one curvature, are also extremely simplified because of the considerations previously exposed. The final result is a reduced model of the vault which confirms the hypotheses proposed (figure 19).
3. – CENTRAL EUROPEAN VAULTS

The great revolution that the development of the Hispanic late Gothic ribbed vault entailed has its direct origin in the arrival of a series of Central European masters to the Burgos and Toledo focal points. Even though the boost caused by this influence in the development of the Hispanic vault work was great and noticeable, they didn’t manage to reach the brilliance and virtuosity of the German designs, where the geometric resources used are of a great complexity and effectiveness. As a counterpoint to the exposition of the analysis of the Hispanic vaults, a paradigmatic case of German vault work is now presented, with one of the tracing methods most used to achieve a systematization and optimization of the work: the *prinzipalbogen*.

A geometric analysis of the ribs of the vault of the Albrechtsburg palace, in Meissen (Germany), which is an example of the vaults called *schlingrippengewölbe*, has been made (figure 20). Some of its ribs are flat curves (their horizontal projection is of a straight line), similar to those seen up to now; however, most of them are three-dimensional curves which are not defined just by their horizontal projection. It is impossible to obtain a vertical projection which gives its real dimension, so it is necessary a systematic and simple method of designing these lines, apparently lacking any geometric reason.

The original sources mention the system called *prinzipalbogen*, or main arch (MÜLLER, WERNER 1990). According to this method, all the ribs of the vault are generated from a unique circumference arch (figure 21): the distance covered in the plan between two points of any rib is moved to a vertical plan where the main arch has been drawn, so the difference in height between these two points is the one determined by their projection over that arch. This way the position of each point of a rib can be known, and is always referred to the *prinzipalbogen*.

To illustrate this aspect, the geometry of three ribs of the vault has been analysed (figure 22). One of them is a flat curve, whereas the other two are three-dimensional curves. After obtaining the spatial position of each one of the points of the joints between the ribs’ voussoirs, two aspects have been verified. First (figure 23), the projection of the three-dimensional curved ribs over the horizontal plan is a circumference arch, i.e. the lay-out of the vault’s plan has been carried out with straight lines and circumference arches. Secondly (figure 24), the distances between the points along the curve where they are located have been measured, and moved to a horizontal line, so as to, later on, situate over the vertical of each one of them, the height at which they’re placed over the ground. The result of placing these points is, in the three cases, a circumference arch, which is no other than the *prinzipalbogen*. In other words, it is proved that each point can be defined by this circle which determines the height of the points depending on the covered length in the plan. This way the geometry of some spatial curved lines apparently arbitrary can be determined. This is a clear example of systematic methodology to generate shapes extremely complex.
4. - CONCLUSIONS

The final Gothic, the one built all along the 15th and 16th centuries, has often been unfairly judged as a Gothic that expands beyond its natural bounds over the Renaissance. The terms of late, decadent or mannerist are those used more frequently to refer to this period and to the architects that played the leading role in it. However, the beauty, the inventiveness and the economy of means which underlies the vaults of the late Gothic make us think that it is precisely at that moment when this style reaches its complete maturity in Europe. Everything seems to point out that it was precisely when it was in its summit that the Gothic was abandoned for a Classicism which represented better the return to the European Greco-Latin roots.

From a constructive point of view, the technological evolution shown is marked by a tendency towards the search for standardization and systematization. At the same time as the designs of the vaults acquired more complex and elaborate features, techniques which allowed making easier their layout and construction were developed.

The systematization of the curvatures of the ribs implies clear benefits. In a direct way, the execution of the voussoirs is somehow simplified, since all the pieces are equal. However, the most significant advantages are related to the organization of the works; on one hand (reducing the necessities of numbering and classification for the storage and placing of the voussoirs), and to the transmission of executing orders from the masters to the stonemasons, on the other hand. The design of the architect had to be translated into instructions for the workers and, even though some of the methods exposed broke the geometric integrity of the arches, the equating of curvatures implied a great advantage for this transmission of orders. This way, the conceptual geometry of the project was adapted to the practical reality of the works, since the differences could be appreciated at first glance. Even more necessary are methods such as the Central European prinzipalbogen, given the geometric complexity of these works (with spatial curves), which makes impossible the drawing of elevations of arches to scale for determine the shape of voussoirs or the construction of centrings.

The derogatory vision that the traditional historiography has shown towards this Gothic of the Renaissance is contradicted with the formidable success it had in almost the whole Europe. If, in addition, one considers the sophisticated use of the geometric resources and the noticeable technological advances, it is not exaggerated to say that the late-medieval vaulting works constitute one of the most brilliant pages in the history of European construction.
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6. - FIGURES

Fig. 1 (left)  Viollet-le-Duc’s interpretation of Villard de Honnecourt’s drawing (A) showing how all the arches of a vault can be drawn from a semi circumference. B, design of a five bosses vault from the semi circumference of the diagonal arch; the result is a vault with its ridge almost horizontal.

Fig. 2 (right)  Summary of the four resources detected in the Spanish Gothic vaults destined to standardize the curvatures.

Fig. 3 (left)  Astorga cathedral, vaults over the chapels of the transept aisle. Rodrigo Gil de Hontañón between 1550 and 1570.

Fig. 4 (right)  Astorga cathedral, vaults over the chapels of the crossings; A, design of the plan and B, of the curvature of their arches.
Fig. 5 (left)  Astorga cathedral, vaults over the chapels of the crossings. Modelling of the vault. Its semicircular diagonal, its flat ridge line, its ribs and bosses can be clearly seen.

Fig. 6 (right)  Segovia cathedral, vault of the cloister. Asymmetrical crossing ribs in this vault built by Juan Guas.

Fig. 7 (left)  Segovia cathedral, vault of the cloister. The vault is built with arches of the same curvature and inclined.

Fig. 8 (right)  Photo of the model made to scale 1:3 in the School of Architecture of Madrid of Juan Guas’ vault.
Fig. 9 (left)  The ribs of an oval vault can be standardized using always the same curvatures; in the figure the tiercerons and the diagonal arch are formed with the same arches.

Fig. 10 (right) San Esteban convent, Salamanca, cloister. Juan de Álava.

Fig. 11 (left) San Esteban convent, Salamanca, cloister. Drawing plan, Fig. A, and curvature of its arches, Fig. B. See the diagonal oval arch and how the horizontal ridge line is obtained by sliding the upper part of the oval over the lower part. All the arches are equal.

Fig. 12 (right) San Esteban convent, Salamanca, cloister. Three-dimensional drawing of the resulting vault.
Fig. 13 (left)  Vault of the elevated choir of the San Esteban convent in Salamanca built by Juan de Álava around 1520.

Fig. 14 (right) Three-dimensional reconstruction of the vault (drawing by Jorge Cerdá).

Fig. 15 (left)  Plan and sections of the vault showing its horizontal longitudinal section and its cross section curved and inclined over the formeret.

Fig. 16 (right) Sketch in which the strategy followed by the architect to standardize the curvatures of the arches is shown. Every vault is traced with the upper part of the diagonal oval arch.
Fig. 17 (left) School of Architecture of Madrid, real scale layouts destined to the construction of a reduced model of the vault at scale 1:3.

Fig. 18 (right) Set-up; the different pieces composing the vault are placed on the centring.

Fig. 19 (left) The reproduction of the complicated vault by Juan de Álava built with only one arch.

Fig. 20 (right) Vault of the Albrechtsburg palace (Meissen, Germany).
Fig. 21 (left) Drawing of the Codex Miniatus 3 manuscript (Österreichische Nationalbibliothek, Vienna), showing the use of the *prinzipalbogen*.

Fig. 22 (right) Analysed ribs of the Albrechtsburg vault (Meissen, Germany).

Fig. 23 (left) Curves obtained from the analysis of the ribs. Horizontal projection.

Fig. 24 (right) Position of the points on an horizontal straight line and at the height measured *in situ*. Verification of the possibility of their placement on the same circumference arch.